Note Title 9/19/2020

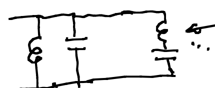
ENEE 610
10/20/20

homework 7
RC PR driving point y, z,
Hurwitz functions

Even/Odd versus Hurwitzian/Skew-Hurwitzian
Rihaards' function & section for synthesis
op-amp curve tracer
exp(-At)
Chain matrix
Constant R circuits
Vo(s)/Vi(s) design

LC, $y(s) \Rightarrow \quad y(s) + y(-s) = 0 \quad (1 \times 1\ y)$ for matrix $Y^T(s) + Y(-s) = 0_{m \times m}$

2nd Foster LC: $y(s) = \frac{k_0}{s} + k_\infty s + \sum \frac{2k_i s}{s^2 + \omega_i^2} \Rightarrow$



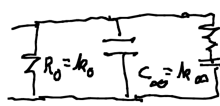
$z_i = \frac{1}{y_i} = L_i s + \frac{1}{C_i s} = \frac{s^2 + \omega_i^2}{2k_i s}$

$L_i = 1/2k_i, \quad C_i = \frac{2k_i}{\omega_i^2}$

$L_0 = 1/k_0 \quad C_\infty = k_\infty$

⇓

RC $\quad y_{RC} = k_0 + k_\infty s + \sum \frac{2k_i s}{s + \omega_i^2}$, $k_0 = y_{RC}(0)$



$R_i = 1/2k_i$, $C_i = \frac{2k_i}{\omega_i^2}$

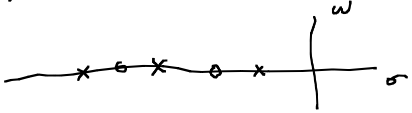
$y_i = \frac{1}{z_i^{RC}} = \frac{1}{\frac{1}{2k_i} + \frac{1}{\frac{2k_i}{\omega_i^2} s}} = \frac{2k_i s}{s + \omega_i^2}$

$R_0 = k_0$, $C_\infty = k_\infty$

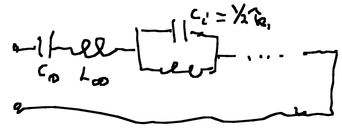
PF $\quad \frac{y_{RC}(s)}{s} = \frac{k_0}{s} + k_\infty + \sum \frac{2k_i}{s + \omega_i^2}$

2nd Foster

Poles are @ $s = -\omega_i^2$, $\omega_i^2 \geq 0$ on negative real axis, zeros in between



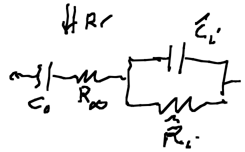
1a.4 Fosters $z_{LC}(s) = \frac{\hat{k}_0}{s} + \hat{k}_\infty s + \sum \frac{2\hat{k}_i s}{s^2+\hat{\omega}_i^2}$



$z_{RC}(s) = \underbrace{\frac{\hat{k}_0}{s}}_{\text{pole @ 0}} + \hat{k}_\infty + \sum \frac{2\hat{k}_i}{s+\hat{\omega}_i^2} \Leftarrow$ partial fraction expansion

pole @ 0
$\hat{k}_0$ is the residue

#RC



$\hat{z}_i = \frac{1}{\hat{y}_i} = \frac{1}{\hat{G}_i + s\hat{C}_i}$

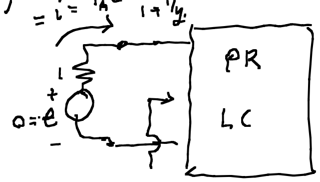
Hurwitz polynomials $P(s) = s^\delta + \dots + a_0 = E(s) + O(s) =$ even + odd.

$\frac{P(s)+P(-s)}{2} + \frac{P(s)-P(-s)}{2} = P(s)$

(no poles of z, y or S in RHP, simple on $j\omega$ axis for z, Z)

$y(s) = \frac{Ev(y(s))}{Od(y(s))}$ or $\frac{Od}{Ev}(s)$

$= i = y_A e = \frac{1}{1+1/y}$



$z = 1/y$, PR

$i \neq 0$ when $e = 0$ if $y_A = \frac{y}{1+y}$ has a pole $\Rightarrow$ a zero of denominator

$y_A = \frac{Od/Ev}{1 + Od/Ev} = \frac{Od}{Ev\underbrace{(Od + Ev)}_{\text{Hurwitz}}}$

Given $P(s) = E(s) + Od(s) = 1 + \left.\frac{Od(P(s))}{\underbrace{E(P(s))}_{y(s)}}\right\}$ LPR if $P(s)$ is Hurwitz so strictly Hurwitz if $\delta[P(s)] = \delta[y(s)]$

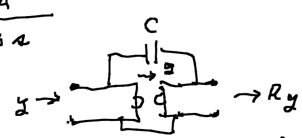
Ex: $P(s) = 3s^3 + 2s^2 + 3s + 4$ is this Hurwitz

$= \underbrace{(2s^2+4)}_{E\nu(s)} + \underbrace{(3s+3s^3)}_{Od(s)}$ $\quad y(s) = \dfrac{3s^3+3s}{2s^2+4}$ is this LPR

1st Cauer (continued fraction expansion about ∞)

$$
\begin{array}{r l}
2s^2+4 \,\big)\; 3s^3+3s & \quad \frac{3}{2}s \\
3s^3+6s & \\
\hline
-3s \,\big)\; 2s^2+4 & \quad -\frac{2}{3}s \\
2s^2 & \\
\hline
4 \,\big)\; -3s & \quad -\frac{3}{4}s
\end{array}
$$

$\Rightarrow y(s) = \frac{3}{2}s + \cfrac{1}{-\frac{2}{3}s + \cfrac{1}{-\frac{3}{4}s}}$ not LPR $\therefore P(s)$ is not Hurwitz



Richards' function

$R(s) = \dfrac{k f(k) - s f(s)}{k f(s) - s f(k)}$ this is PR if $f(s)$ is PR & $k > 0$

$\delta / f(s)$

the pole @ $s = k$ cancels
also a pole @ $s = -k$ if $\left. E\nu\, f(s) \right|_{s=k} = 0$ this is true for LPR $f(s)$ as they are odd for all $k > 0$

then $\delta[R] = \delta[f(s)] - 1$

Ex: $f(s) = y(s) = s + \frac{2}{s} = \frac{s^2+2}{s}$ choose $k = 1$, $y(k) = 1 + \frac{2}{1} = 3$

$\dfrac{R(s)}{3} = \dfrac{1 \cdot 3 - s\, y(s)}{k\, y(s) - s \cdot 3} = \dfrac{3 - \frac{s^3+2s}{s}}{1\left(\frac{s^2+2}{s}\right) - 3s} = \dfrac{3s - s^3 - 2s}{s^2 + 2 - 3s^2} = \dfrac{-s^3 + s}{-2s^2 + 2} = \dfrac{s(-s^2+1)}{+2(-s^2+1)}$

$= s/2$ cancelled $s^2 - 1 = s^2 - k^2$

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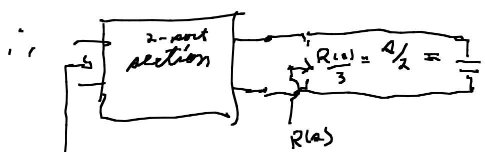
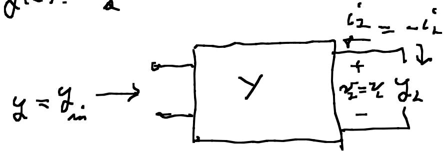


$Y(s) = \frac{s^2+2}{s}$



$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$-y_L v_2 = i_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow -(y_L + y_{22}) v_2 = y_{21} v_1 \qquad \Delta_y = y_{11} y_{22} - y_{21} y_{12}$$

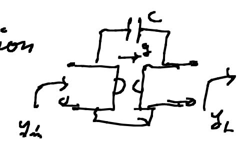
$$i_1 = \left[y_{11} - \frac{y_{12} y_{21}}{y_{22} + y_L} \right] v_1 \Rightarrow y_{in} = \frac{\Delta_y + y_{11} y_L}{y_{22} + y_L}$$

solve y_L vs y_{in} $\Rightarrow$ $y_{in} y_{22} + y_{in} y_L = \Delta_y + y_{11} y_L \Rightarrow (y_{in} - y_{11}) y_L = \Delta_y - y_{in} y_{22}$

$$y_L = \frac{\Delta_y - y_{22} y_{in}}{y_{in} - y_{11}}$$

for the Richards' section

$$Y = \begin{bmatrix} sc & -sc - g \\ -sc + g & sc \end{bmatrix}$$



$$\Delta_y = g^2$$

$$y_L\Big|_{\text{Richards}} = \frac{g^2 - sc\, y_{in}(s)}{y_{in}(s) - sc} = \frac{g\left(1 - \frac{sc}{g} \frac{y_{in}(s)}{g}\right)}{1/g\,(y_{in}(s) - sc)} = g \left(\frac{1 - \frac{sc}{g} \frac{y_{in}(s)}{g}}{\left(\frac{y_{in}(s)}{g} - \frac{sc}{g}\right)} \right) \Rightarrow \frac{y_L}{g} = \left(\frac{1 - \frac{sc}{g} \cdot \frac{y_{in}(s)}{g}}{\frac{y_{in}(s)}{g} - \frac{sc}{g}} \right)$$

$$\frac{R}{y'(k)} = \frac{k\,y(k) - a\,y(a)}{k\,y'(a) - a\,y'(k)} = \frac{k\,y(k)\left(1 - \frac{a}{k}\frac{y(a)}{y(k)}\right)}{k\,y'(k)\left(\frac{y'(a)}{y'(k)} - \frac{a}{k}\right)} = \frac{\left(1 - \frac{a}{k}\cdot\frac{y(a)}{y(k)}\right)}{\left(\frac{y'(a)}{y'(k)} - \frac{a}{k}\right)} = \Rightarrow \quad k = g/c, \quad g = y(k), \quad c = \frac{g}{k} = \frac{y(k)}{k}$$

given $y = y_{in}$ choose k for $E_0[y(a)] = 0$ then set $g = y(k)$, $c = \frac{y_0(k)}{k}$

then $y_2(a)$ has $a^2 - k^2$ cancels in numerator & denominator

so $\delta[y_2] = \delta[y] - 1$